THE ROYAL AERONAUTICAL SOCIETY

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS

## MONTHLY NOTICES

MARCH, 1942.

### Annual and Special General Meetings.

Below are printed formal notices of the Annual and Special General Meetings. On account of the shortage of paper further notices will not be sent to members.

### Annual General Meeting.

NOTICE IS HEREBY GIVEN that the Annual General Meeting of the Royal Aeronautical Society, with which is incorporated the Institution of Aeronautical Engineers, will be held on Saturday, March 28th, 1942, at 2.30 p.m., at the Offices of the Society, No. 4, Hamilton Place, London, W.1.

AGENDA.

1. To read the Notice convening the Meeting.
2. To receive and deliberate upon the Report of the Council on the state of the Society and the Balance Sheets of Aerial Science Limited and Aeronautical Trusts Limited for the year ended December 31st, 1941.
3. To receive the nominations for Council for the years 1942-1944.
4. To elect the Auditors for the ensuing year for Aerial Science Limited and Aeronautical Trusts Limited.
5. To consider any other business.

By Order of the Council,

J. LAURENCE PRITCHARD,

*Secretary.*

### Special General Meeting.

*February 1st*, 1942.

NOTICE IS HEREBY GIVEN that a Special General Meeting, under Rule 100, of the Royal Aeronautical Society, with which is incorporated the Institution of Aeronautical Engineers, will be held on Saturday, March 28th, 1942, at 3.30 p.m., in the Offices of the Society at No. 4, Hamilton Place, London, W.1, to consider the following alterations and additions to the Rules:—

Rules numbered 3-8*c* inclusive to be re-numbered and to read as follows:—

RULE 3. Fellows shall comprise every person who was on the Register on March 28th, 1942, as a Fellow of the Royal Aeronautical Society, with which is incorporated the Institution of Aeronautical Engineers, and every person thereafter elected or transferred into the class of Fellow.

RULE 4. Election to Fellowship will be made annually by the Council for announcement at the Annual General Meeting of the Society in March.

**Annual Subscriptions.**

Members are reminded that their subscriptions are due on January 1st, 1942. The following are the current rates:—

| | Home. £ s. d. | Abroad. £ s. d. |
|---|---|---|
| Founder Members ... ... ... | 2 2 0 | 2 2 0 |
| Fellows ... ... ... ... | 4 4 0 | 3 3 0 |
| Associate Fellows ... ... ... | 3 3 0 | 2 2 0 |
| Associates ... ... ... ... | *2 2 0 | *2 2 0 |
| Graduates (age 21-25) ... ... | 2 2 0 | 2 2 0 |
| ,, (age 26-28) ... ... | 2 12 6 | 2 12 6 |
| Students ... ... ... ... | 1 1 0 | 1 1 0 |
| Companions .. ... ... ... | 2 2 0 | 2 2 0 |

* £1 1s. 0d. without JOURNAL.

The attention of members is drawn to Rule 55, as follows:—

"Any Fellow, Associate Fellow, Associate, Graduate, Student, Companion or Founder Member, whose subscription shall not have been paid before the first day of April, shall be in arrears of subscription and shall not be entitled to attend or vote at any meeting or to receive any publications to which he would be otherwise entitled."

**Associate Fellowship Examinations.**

The next Associate Fellowship examination will be held in the Offices of the Society in May, 1942. Entries should be in by March 31st, 1942, for the May examination.

The following is a list of successful candidates in the December, 1941, Associate Fellowship Examinations:—

| | |
|---|---|
| C. L. Brokenshaw ... | Pure Mathematics. |
| C. Y. Carayan ... | Aircraft Materials. |
| A. F. Carr ... ... | Strength of Materials and Theory of Structures. |
| P. Farr ... ... | Strength of Materials and Theory of Structures (First Place). |
| G. W. S. Griffith ... | Theory of Internal Combustion Engines. |
| L. S. Greenland ... | Theory of Internal Combustion Engines. |
| E. G. Havard ... ... | Applied Mathematics. |
| E. J. Hellier ... ... | Applied Mathematics (First Place—Tie). |
| E. G. Jones ... ... | Pure Mathematics. |
| J. P. Machado... ... | Pure Mathematics.<br>Design (Aircraft).<br>Theory of Machines. |
| R. H. P. Nott... ... | Design (Aero Engines).<br>Theory of Internal Combustion Engines. |
| G. B. Saksena ... | Design (Aero Engines).<br>Theory of Internal Combustion Engines. |
| I. H. G. Silander ... | Applied Mathematics.<br>Strength of Materials and Theory of Structures.<br>Design (Aircraft). |
| R. T. Weatherstone ... | Theory of Internal Combustion Engines. |
| D. C. Whittley ... | Pure Mathematics (First Place).<br>Theory of Machines (First Place). |
| H. Wilson ... ... | Applied Mathematics (First Place—Tie).<br>Design (Aero Engines) (First Place).<br>Theory of Internal Combustion Engines (First Place). |

**Baden-Powell Memorial Prize.**

The Baden-Powell Memorial Prize has been awarded to Mr. H. Wilson who was considered by the Examiners to be the best student in the December, 1941, Associate Fellowship Examinations.

**Election of Members.**

*Fellow.*—Frank Charles Lynam (from Associate Fellow).

*Associate Fellows.*—John Edward Adamson (from Student), Denis Robert Hayes Dickinson (from Student), George Robert Edwards, Jacobus Johannes Gerritsen, Geoffrey Kingslake, Ivor Lusty, Charles Leslie Old, Robert McIntyre, Haydn Templeton.

*Associates.*—Sydney Armishaw, Joseph Henry Ernest Heath Billingsley, Sidney Herbert Bostock, Alfred Thomas Day, John James Hedges, Thomas Gordon Lloyd, Wilfred Morgan Protheroe, Henry Percy Thornton.

*Graduates.*—Robert William Butler, Peter Farr, Jack Francis, George Hills Forster, Frank Taylor, Harry Udall (from Student), Frederick Westall, William Young Wilson.

*Students.*—James Mitchell Bryson, Colin Elliot Child, Ian James Alexander Cruickshanks, Derrick Orme Dooyewaard, Francis William Free, Reginald Bertram Glanville-Wood, Robert Stanley Hammond, Jeffrey Hammerton Harwood, Leonard William Kilbourne, Robert Elkan Landau, Lyndon George McFarlane, Ronald George Manley, John Mowbray Morton, Peter James Nyland, Timothy Patrick O'Regan, George Pearson, Kenneth Hugh Prichard, Barrie Raymond Robin, John George Romeril, John Marler Sloper, David Alan Ernest Smith, Kenneth William Toulson, Thomas Edward Voss, David Walker, James Carey Wallin.

*Companions.*—Archibald Buchanan Barbour, Oladigbolu Coker, Adolph H. Emden, Charles Leslie Ross, Harold Victor Stinson, Baruch Harold Wood.

**Graduates' and Students' Section.**

Members of the Graduates' and Students' Section are invited to attend the following meeting of the Graduates' Section of the Institution of Automobile Engineers, 12, Hobart Place, S.W.1:—

Saturday, May 16th, 3 p.m.—Lecture on Plastics by Mr. Curwen, Editor of "Plastics."

**Acknowledgments.**

The Council acknowledge with grateful thanks the gift of JOURNALS from the following members:—

Miss L. Chitty, Fellow.
Air Commodore H. M. Cave-Browne-Cave, Fellow.
W. Garrow-Fisher, Associate Fellow.
Miss A. D. Betts, Companion.

**Additions to the Library.**

Pamphlets in italics, with location reference following in brackets.
Books marked * may not be taken out on loan.

B.a.250.—Bombers. By C. G. Grey. Faber and Faber. 1941. 6/-.

*B.g.110.—*Aircraft Recognition Test.* (*2nd Ed.*) *By J. Hay Stevens.* *A.T.C. Gazette.* 1942. 4*d.* (PB.5-21.)

BB.b.92.—*Weight Control—Aircraft Design Problem. E. J. Foley. Soc. of Aeron. Weight Engineers, U.S.A. Paper No.* 12. (Y.34.)

BB.b.93 and 94.—Aircraft Stressing. By W. L. Morse. Chas. Griffin and Co. 1941. 18/-. (Two copies).

BB.d.27.—*Calculations of Landing Gear and Hydraulic System Weights. By Harold W. Adams (Soc. of Aeron. Weight Engineers. Paper No. 10.)* (Y.34.)

D.f.36 and 37.—*Notes on Fire Prevention (1 and 2). Air Ministry Official Publications (I.M. 1004 and 1008).* 1919-1920. (Y.12.a.1.)

EE.d.8.—*Carburettor Jet Sizes, Mark 2. Air Ministry Official Publication. (T.D.I.576.)* 1919. (Y.7.a.1.)

EE.e.18 and 19.—*The History of the Magneto. By Frederick R. Simms. Privately printed.* 1940. (Y.7.a.11 and 12.) (Two copies.)

EE.f.48.—High Speed Diesel Engines. (4th Ed.) By A. W. Judge. Chapman and Hall. 1942. 25/-.

EE.f.49.—The Modern Diesel. (6th Ed.) Published by Iliffe and Sons. 1941. 5/-.

F.a.8-19.—*Air Ministry Official Publications.* (1918-1922.) (Y.7.a.1—Envelope.)

I.C.675. Petrol Pump.
I.M.1010. Vickers Centrifugal Pump, Mark II.
T.D.I.577. Austin " Glandless " Petrol Pump.
T.D.I.561. Le Bozec Non-Return Valves.
T.D.I.569. Petrol Cocks.
A.P.902. Standard Fuel Filters.
A.P.813. Vickers Petrol Relief Valve, Mark II.
C.I.M.730. Petrol Systems. (Two copies.)
C.I.M.731. Badin Vacuum Feed.
C.I.M.723. " Imber " Self-Sealing Tank.
T.C.650. Lopdell Hose Clip. (Y.7.c.1.)

F.b.34.—*Evolution in the Petroleum Industry. By James Kewley. Royal Soc. of Arts. Paper read 21/1/42.* (Y.7.c.14.)

G.a.52 and 53.—*Notes on an X-Ray Investigation into Cold-Rolled Magnesium Alloy Sheet. By Leopold Fromer. Institute of Metals. (Reprint.)* 1941. (PG.1-16 and 17.) Two copies.

*G.b.3 and 4.—*British Standards Institution: Specifications.* 1941-1942. (1/- each.)

A.17. Machined Hexagonal-Headed Bolts.
A.18. Machined Hexagonal Nuts. (Aluminium Alloy.)
3S.21. " 20 " Carbon Steel (suitable for Welding).
3T.26. 20-ton Steel Tubes (suitable for Welding).

G.d.13-15.—*Air Ministry Official Publications.* (1919.) (Y.13.a.1.)

T.D.I.570. Protection of Steel Parts of Seaplanes against Corrosion.
T.D.I.571. Protection against Corrosion of Steel Parts on Aeroplanes.
T.D.I.572. Japanning and Stove Enamelling of Metal Fittings.

G.e.A.74.—*Air Ministry Official Publication.* (1919.) (Y.13.c.A.)

T.D.I.573. Corrosion of Aluminium and its Alloys as Used in Aircraft.

G.e.B.18.—*Pipe Line Metal Coupling. (A.P.854.) Air Ministry.* 1921. (Y.13.c.C.)

G.e.E.53.—*Steel Tubes, Tube Manipulation and Tubular Structure for Aircraft. By W. W. and A. G. Hackett. (Air Ministry Reprint from "The Aeronautical Journal," Vol. XXII, No. 88.) (I.M.1016.)* (Y.13.c.S.) 1920.

G.e.F.11.—Rubber as an Engineering Material. British Tyre and Rubber Co. 1941.

*G.e.F.12.—*Rubber Data for the Engineer. (Revolving Card.)* 1942. (Y.13.c.R.)

G.e.F.13-16.—*Air Ministry Official Publications.* 1918-1920. (Y.13.c.R.)

T.D.I.540. Petrol Resisting Rubber Tubing.

I.M.1000. The " Parnacott " Flexible Rubber Connection for Pipe Work.

I.M.1005. Petrol Resisting Tubing.

I.M.1911. Flexible Joints and Couplings.

G.f.23 and 24.—*Air Ministry Official Publications.* 1918-1919. (Y.13.b.1 and 1a.)

T.D.I.553. Instructions for Soft Soldering.

C.I.M.710. Methods of Fixing Tubes to Sockets.

H.b.84.—*Reciprocal Recognition of Certificates of Airworthiness for Imported Aircraft. U.S. Govt. Printing Office.* 1934. (PH.1.b.6.)

I.a.25.—*Elementary Aircraft Mathematics. By J. B. Channon and A. McLeish Smith. Longmans, Green and Co.* 1942. 1/6. (PI.1-8.)

I.a.26.—Mathematics for Technical Students. (Part III.) By A. Geary, H. V. Lowry and H. A. Hayden. Longmans, Green and Co. 1941. 8/-.

L.d.69 and 70.—Air Navigation: Some Problems and Their Solution. By M. J. Hearley. Longmans, Green and Co. 1942. 5/-. (Two copies.)

L.d.71.—Astrographics: or First Steps in Navigation by the Stars. By Frank Debenham. W. Heffer, Cambridge. 1942. 7/6.

L.d.72.—Teach Yourself Air Navigation. By " Kaspar." English Universities Press. 1942. 2/6.

M.b.45.—*Ministry of Munitions Official Specification.* 1918. (Y.18.a.20a.) T.D.I.505A. W/T " Earth " in Aircraft.

M.c.52.—Direction Finding Handbook for Wireless Operators. By W. E. Crook. Sir Isaac Pitman and Sons. 1942. 3/6.

R.e.48.—*Methods of Handicapping: King's Cup Race and Croydon Stakes.* 1925; *with blueprint showing results. Royal Aircraft Establishment, Farnborough.* (Y.11.iv.a.1a.)

R.f.73.—The Birth of Flight. (An Anthology.) By Hartley K. Cook. Allen and Unwin. 1942. 7/6.

R.f.74.—Air Power and Civilization. By M. J. B. Davy. Allen and Unwin. 1941. 8/6.

T.a.95 and 96.—*Frederick R. Simms. (Autobiographical Data.) (Stencil.)* 1940. (Y.5/S.) (Two copies.)

UC.3/1.—*Engineering Institute of Canada: Engineering Journal, Aeronautical Section Reprint No. 10, November, 1941. Containing articles by Eliz. MacGill, J. A. Wilson, A. T. E. Wanek and Ewan D. Boyd.* (File Case.)

V.13/1-3.—*Society of Aeronautical Weight Engineers, U.S.A.* (Box File Y.34.) Summary of Minutes: S.A.W.E. National Meeting, 1941. Nineteen Forty-One Year Book, Vol. 1, No. 1. Circular Letters Nos. 11, 26, 29, 33, 34, 35, 39, 40, 41, 42 and 43.

*X.b.141, 142.—Annuaire de l'Aviation Yougoslave. Edited by R. P. Givkovitch and D. D. Loutchitch. " Jugoslovenska Krila," Belgrade. Two Vols., 1936 and 1940. (Text in Serbian: French sub-titles to chapter headings, illustrations, etc.)

J. LAURENCE PRITCHARD, *Secretary and Editor.*

# SUPERSONIC FLOW IN TURBINES AND COMPRESSORS.*

K. W. SORG.

"*Forschung.*" *Vol.* 10, *No.* 6, *Nov.-Dec.*, 1939, *pp.* 270-285.

Translated by L. J. GOODLET.

Whereas a body of theoretical research is available on the conditions of supersonic flow through Laval nozzles and around aerofoils, such is lacking for turbo-machinery in general, with the exception of some researches by Stodola,† confined to axial steam turbines. The present paper investigates some important questions relating to the flow at supersonic velocity through centrifugal machines of the general type.

NOTATION.

| | | |
|---|---|---|
| $a$ | [m./sec.] | = velocity of sound. |
| $a^*$ | [m./sec.] | = velocity of sound in the presence of influx or efflux of mechanical energy. |
| $p$ | [kg./m.$^2$] | = pressure. |
| $\rho$ | [kg. s$^2$/m.$^4$] | = density. |
| $F_c$ or $F_w$ | [m.$^2$] | = cross-section perpendicular to the absolute or relative flow. |
| $G$ | [kg./sec.] | = weight of fluid flow per second. |
| $v$ | [cbm./sec.] | = specific volume. |
| $s$ | [m.] | = absolute flow path. |
| $i$ | [kilocal./kg.] | = heat content (enthalpy). |
| $L$ | [mkg./kg.] | = external work. |
| $A$ | [kcal./mkg.] | = mechanical equivalent of heat. |
| $g$ | [m./sec.$^2$] | = acceleration of gravity. |
| $c$ | [m./sec.] | = absolute velocity |
| $w$ | [m./sec.] | = relative velocity. |
| $c_u$ or $c_r$ | [m./sec.] | = peripheral or radial component of the absolute velocity. |
| $u$ | [m./sec.] | = peripheral velocity of a point on the rotor. |
| $k$ | | = a dimensionless constant. |
| $\sigma$ | | = a small dimensionless factor. |

## I. THE PROBLEM AND THE FUNDAMENTAL EQUATIONS

Two salient facts characterise the steady flow of compressible fluids (gas or vapour). Firstly, the transmission from subsonic to supersonic velocity is related to a minimum cross-section in which the velocity of sound corresponding to the conditions in the section prevails; secondly, compression shocks can originate in a flow with supersonic velocity. The local "sonic velocity" $a$ is then defined by the expression:—

$$a^2 = \frac{d\rho}{dp} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

* R.T.P. Translation. Published by permission of the Ministry of Aircraft Production.

† A. Stodola. "Dampf-und Gasturbinen" ("Steam and Gas Turbines"). 6th Ed., Berlin 1924, pp. 131 *et seq*, p. 809 *et seq*.

The flow through a centrifugal rotor (turbine) is in principle unsteady, which is explained by the fact that it either receives energy "from outside," or imparts energy "to the outside." The question now is: What are the conditions for the existence of a minimum cross-section, and the occurrence of compression shocks in turbine rotors? For the purpose of a first, simplified examination of this question, it is proposed to use the unidimensional theory of turbine rotors, according to which the number of blades is assumed to be infinite, and the unsteady character of the flow through the rotor consequently vanishes. The flow will therefore be regarded as essentially steady, but accompanied by an exchange of energy with the surrounding medium.

Three equations are then available for the mathematical analysis of the problem.

Firstly, the equation of continuity:—

$$F = G\left(\frac{v}{c}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Reduction of the differential quotient $dF/ds$ to zero produces the following minimum condition for the cross-section $F$:—

$$\frac{c}{v} = \frac{dc}{dv} = \frac{(dc/ds)}{(dv/ds)} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Secondly, the equation of energy:—

$$di + A\frac{c}{g}\,dc = A\,dL \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thirdly, the equation of state. This is based on the adiabatic:—

$$p = \text{const.}\ \rho^{\eta} = p_0\left(\frac{\rho}{\rho_0}\right)^{\eta}, \text{ or } pv^{\eta} = p_0 v_0^{\eta} \quad . \quad . \quad (5), (6)$$

Then

$$a^2 = \eta\left(\frac{p}{\rho}\right) = \eta g p v\,; \frac{dp}{dv} = -\eta\left(\frac{p}{v}\right);\ dv = -\frac{(v\,dp)}{(\eta p)}. \quad . \quad . \quad (7)$$

Since thus no addition or abstraction of heat is permissible ($dQ = 0$), hence:—

$$di - Av\,dp = 0 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Combining Eqs. 4 and 8, and taking $\rho = 1/gv$:—

$$\frac{dp}{\rho} + c\,dc - g\,dL = 0 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Hence by Eq. 3, and in consideration of Eq. 7, the *equation of condition for an extremum of the cross-section becomes*:—

$$\frac{c^2}{a^2} = 1 - \frac{g\,dL}{(dp/\rho)} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where $a^2 = dp/d\rho$.

The term $c/a$ is known in gas dynamics as the Mach number. For a flow without energy influx, the minimum of the cross-section occurs when the Mach number $c/a = 1$. With the addition or abstraction of energy, a minimum of the cross-section can occur at Mach numbers either greater or less than unity, or not at all. This will depend on the ratio of the kinetic or pressure energy of the flow, to the energy input. It is therefore necessary to investigate the variation of the pressure or kinetic energy consequent on the influx of mechanical energy.

## II. General Application to Turbine Rotors.

Firstly, directly following Eq. 10, the absolute flow and the cross-section $F_c$ normal to it, will be examined.

For a turbine rotor:—

$$dL = \left(\frac{1}{g}\right) d\,(c_u u) \quad . \quad . \quad . \quad . \quad . \quad (11)$$

whence by Eq. 10:—

$$\frac{a^2}{c^2} = 1 - \frac{d\,(c_u u)}{d\,(c^2/2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

For an absolute flow through a turbine the occurrence of an extremum of the cross-section, and the corresponding ratio $a/c$, depend solely on the proportion of mechanical energy input in the form of an increment of kinetic energy. Fig. 1 gives a graphical representation of this relationship. $d\,(c_u u)/d\,(c^2/2) = 1$ is the upper limit of an extremum of the cross-section, corresponding to an impulse rotor, since the flow through rotor is expressed by Eq. 9 as:—

$$\frac{dp}{\rho} + c\,dc - d\,(c_u u) = 0 \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and consequently, $dp/\rho = 0$, for $d\,(c_u u)/d\,(c^2/2) = 1$.

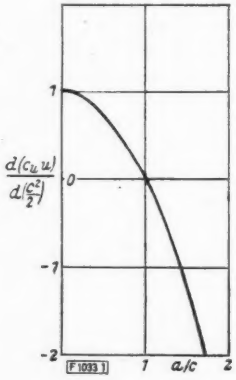


FIG. 1.

*Extremum of the cross-section $F_c$ (normal to the absolute flow), according to Eq. 12.*

An extremum of the cross-section is only possible if pressure and velocity gradients are opposed in direction, as will be seen from the following Eq. 14:—

$$\frac{a^2}{c^2} = -\frac{(dp/\rho)}{c\,dc} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

which is obtained by introducing Eq. 13 into Eq. 12. For instance, an extremum of the cross-section $F_c$ can only occur in a turbine rotor at continuously increasing absolute velocity, if the pressure in the flow through the rotor continuously decreases. For the case of continuously decreasing absolute velocity, the occurrence of an extremum of the cross-section is only possible if the pressure in the flow through the wheel, continuously increases. Qualitatively, this corresponds to the case of a flow through a stationary channel without influx of energy, *e.g.*, through a Laval nozzle. The rotor flow differs, however, in that the extremum will not be at the point where the flow velocity $c$ equals the local sonic velocity $a$.

The examination will now be extended to the relative flow, and the cross-section $F_w$ normal thereto.

In this case the equation of condition for an extremum of $F_w$ is in place of Eq. 3:—

$$\frac{w}{v} = \frac{(dw/dx)}{(dv/dx)} = \frac{dw}{dv} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The relative velocity can be substituted for the absolute velocity in Eq. 13, by means of the expression $w^2=c^2+u^2-2c_u u$, when Eq. 13 assumes the following form :—

$$\frac{dp}{\rho}+wdw-udu=0 \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Introducing Eq. 16 in Eq. 15, and remembering that Eq. 7 also applies, it follows that :—

$$\frac{w}{v}=\frac{(dp/\rho-udu)\,\eta p}{wvdp}$$

Let the sonic velocity be introduced now as the velocity of reference :—

$$a=\sqrt{\left(\frac{dp}{d\rho}\right)}=\sqrt{\left(\frac{\eta p}{\rho}\right)}$$

($\eta$=adiabatic index)

whence :—

$$\frac{w^2}{a^2}=1-\frac{d\,(u^2/2)}{(dp/\rho)}=-\frac{d\,(w^2/2)}{(dp/\rho)} \quad . \quad . \quad . \quad . \quad (17)$$

or

$$\frac{a^2}{w^2}=1-\frac{d\,(u^2/2)}{d\,(w^2/2)} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The graphical representation of Eq. 18 is given by Fig. 1, if the abscissæ are taken as $a/w$ instead of $a/c$, and the ordinates as $d\,(u^2/2)/d\,(w^2/2)$ instead of $d\,(c_u u)/d\,(c^2/2)$. $d\,(u^2/2)/d\,(w^2/2)$ is the ratio of the pressure variation produced by the effect of centrifugal force to the pressure variation produced by change in the relative velocity. $d\,(u^2/2)/d\,(w^2/2)=1$ is also in this case, the limit for the possibility of an extremum, and similarly indicates an influx of energy at constant pressure, as can be recognised from Eq. 16. In accordance with Eq. 17, the general condition for the occurrence of an extremum of $F_w$, is that the gradients of pressure and velocity should be opposed in direction. This condition corresponds exactly to the conditions applying for the extremum of the cross-section $F_c$, and for the case of a stationary channel without influx of energy. Similarly, it will be seen by Eqs. 17 or 18, that in the general case the cross-section $F_w$ cannot have an extremum when the relative velocity $w$ is equal to the local sonic velocity $a$. It is consequently not permissible to assume that the behaviour of the relative velocity in a rotating wheel (rotor) on attaining or exceeding the velocity of sound, is the same as the behaviour of the absolute velocity in channels without the influx of energy. Furthermore, it follows from the preceding exposition, that in all cases where an extremum cannot occur—*i.e.*, when $d\,(c_u u)/d\,(c^2/2)>1$ and $d\,(u^2/2)/d\,(w^2/2)>1$—both absolute and relative velocities can attain supersonic magnitude without any restriction of the cross-sectional area of the flow. For $d\,(c_u u)/d\,(c^2/2)=1$ this appears obvious, since the entire input energy is converted into increased kinetic energy, the pressure and thus the specific volume, remaining constant. An extremum of the cross-section is therefore obviously impossible, since the extremum as such, is produced solely by the interaction of the increments of velocity and volume. For $u=0$, *i.e.*, for a stationary channel both Eq. 12 and Eq. 18 furnish $a/c=1$ and $a/w=1$ respectively; of course, if $u=0$, then $w=0$.

For the case that $c_u u=$const., *i.e.*, that no exchange of mechanical energy takes place between the rotor and the flow, the gas passing through the rotor without deflection, Eq. 12 indicates the necessary occurrence of an extremum of the cross-section for $a/c=1$. Eq. 18 does not yet, however, furnish any indication of the necessity of an extremum for $F_w$.

Eq. 18 conveys considerable information on the flow through axial wheels (rotors), which will be dealt with later.

The values defining the blade-form will now be introduced, not only for the equations of condition of the extremum, but also for $dF_c/ds$ and $dF_w/dx$, in order to obtain information as to the development of the cross-section. This will first be done for the cross-section $F_c$.

The equation of continuity can be written in the form :—

$$\frac{dF_c}{F_c}+\frac{dc_z}{c}+\frac{d\rho}{\rho}=0 \quad . \quad . \quad . \quad . \quad . \quad (19)$$

If the velocity of sound $a=\sqrt{(dp/d\rho)}$ is again taken as the velocity of reference, Eq. 13 assumes the form :—

$$a^2\left(\frac{d\rho}{\rho}\right)+cdc-d\,(c_u u-u)=0 \quad . \quad . \quad . \quad . \quad (20)$$

Introducing Eq. 20 into Eq. 19 :—

$$\frac{dF_c}{F_c}+\frac{dc}{c}+\frac{d\,(c_u u)}{a^2}-\frac{cdc}{a^2}=0$$

whence :—

$$\frac{dF_c}{du}=-\frac{F_c}{c}\left\{\frac{dc}{du}+\left(\frac{c}{a^2}\right)\frac{d\,(c_u u)}{du}-\left(\frac{c^2}{a^2}\right)\cdot\left(\frac{dc}{du}\right)\right\}$$

Taking into consideration the equation of continuity, and the fact that for an adiabatic change of state in the rotor, $a^2=\eta p/\rho$, into consideration :—

$$\frac{dF_c}{du}=-\left(\frac{G}{g\eta p}\right)\left\{\left(\frac{dc}{du}\right)\left(\frac{a^2}{c^2}-1\right)+\frac{d\,(c_u u)}{cdu}\right\} \quad . \quad . \quad (21)$$

Since $c^2=c_r^2+c_u^2$, $dc/du$ can also be written as :—

$$\frac{dc}{du}=\frac{dc^2}{2cdu}=\left(\frac{c_r}{c}\right)\cdot\left(\frac{dc_r}{du}\right)+\left(\frac{c_u}{c}\right)\cdot\left(\frac{dc_u}{du}\right)$$

Ultimately, by introducing this expression in Eq. 21 :—

$$\frac{dF_c}{du}=-\left(\frac{G}{g\eta p}\right)\left\{\left[\left(\frac{c_r}{c}\right)\cdot\left(\frac{dc_r}{du}\right)+\left(\frac{c_u}{c}\right)\cdot\left(\frac{dc_u}{du}\right)\right]\left(\frac{a^2}{c^2}-1\right)+\left(\frac{u}{c}\right)\cdot\left(\frac{dc_u}{du}\right)+\left(\frac{c_u}{c}\right)\right\} \quad (22)$$

This Eq. 22 now allows conclusions on the development of the cross-section to be drawn for the majority of cases. An extremum of the cross-section will occur when :—

$$\frac{a^2}{c^2}=1-\left\{\frac{(u/c)}{(c_r/c)}\cdot\frac{(dcu/du)}{(dc_r/du)}+\frac{c_u/c}{c_u dc_u/cdu}\right. \quad . \quad . \quad . \quad (23)$$

It is, of course, easily seen that Eqs. 12 and 23 are identical.

For the relative cross-section $F_w$, by suitable transformation :—

$$\frac{dF_w}{du}=-\left(\frac{G}{gpw^2}\right)\left\{\left(\frac{dw}{du}\right)\left(1-\frac{w^2}{a^2}\right)+\frac{uw}{a^2}\right\} . \quad . \quad . \quad (24)$$

The condition for an extremum of the cross-section $F_w$ is obtained by Eq. 25 as :—

$$\frac{a^2}{w^2}=1-\left(\frac{u}{w}\right)\cdot\left(\frac{du}{dw}\right) . \quad . \quad . \quad . \quad . \quad (25)$$

Eq. 25 is identical with Eq. 18 and therefore does not yet admit of any conclusions excepting those furnished by Eq. 14.

For certain purposes it is useful to combine the relative equation of continuity with the absolute equation of energy. In such case :—

$$\frac{dF_w}{du}=-\left(\frac{G}{g\eta p}\right)\left\{\left(\frac{a^2}{w^2}\right)\cdot\left(\frac{dw}{du}\right)+\left(\frac{dc_u}{du}\right)\left(\frac{u}{w}-\frac{c_u}{w}\right)-\left(\frac{c_r}{w}\right)\cdot\left(\frac{dc_r}{du}\right)+\frac{c_u}{w}\right\} \quad (26)$$

and the equation of condition for an extremum of the cross-section $F_w$ becomes :—

$$\frac{a^2}{w^2}=1-\frac{1}{\left(1-\frac{c_u}{u}\right)\left(1+\frac{dc_u}{du}\right)+\frac{c_r}{u}\cdot\frac{dc_r}{du}} \quad . \quad . \quad . \quad (27)$$

## III. Examination of the Development of the Cross-Section for Varying Forms of Rotor and Blade.

### 1. Radial Rotor with Purely Radial Blades.

Case " a ":—

$$c_u = u\,;\ \frac{dc_u}{du} = 1\,;\ \frac{dc_r}{du} = 0 \text{ (constant radial component)}$$

Application of Eq. 22 furnishes:—

$$\frac{dF_c}{du} = -\left(\frac{G}{g\eta p}\right)\left\{\left(\frac{u}{c}\right)\left(\frac{a^2}{c^2} - 1\right) + \frac{u}{c} + \frac{u}{c}\right\} = -\left(\frac{G}{g\eta p}\right)\left\{\left(\frac{u}{c}\right)\left(\frac{a^2}{c^2} + 1\right)\right\}$$

from which it is evident that in this case the cross-section $F_c$ continuously decreases. An extremum of the section $F_c$ is therefore impossible. The same applies for the cross-section $F_w$, as can be seen from Eqs. 26 and 27. Taking into consideration that $w = c_r$, Eq. 26 shows that:—

$$\frac{dF_w}{du} = -\left(\frac{gG}{g\eta p}\right)\cdot\left(\frac{c_u}{w}\right)$$

*i.e.*, the cross-section continuously decreases. This agrees, of course, with Eqs. 14 and 23. *E.g.*, in the case of a turbo-compressor, the absolute velocities, under the assumptions made, continuously decrease. The increment of energy of the absolute velocity, however, is less than the energy influx at any point in

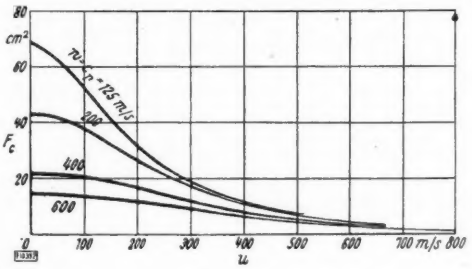


FIG. 2.

*Distribution of the cross-section $F_c$ (normal to the absolute flow), dependent on the peripheral velocity u.*

the rotor channel, and therefore the pressure must also continuously increase. The pressure and velocity gradients consequently have identical direction, and no extremum of the cross-section can occur. Figs. 2-5 show some numerical examples of the actual development of the cross-section, representing the development of $F_c$ and $F_w$ against $u$ and $c/a$ or $w/a$ respectively, for a number of values of $c_r^2$. The numerical calculation is based on a centrifugal air compressor with a delivery of $G = 1$ kg.$-s$, and constants of $p_o = 1$ atm. absolute pressure, $t_o = 20°C.$, and $c_r = \text{const.} = c_{ro}$. The development of conditions in the rotor is assumed to be without losses, *i.e.*, adiabatic. The increase of enthalpy in the rotor according to Eq. 4 is then:—

$$\Delta i = \left(\frac{A}{g}\right)\left\{\Delta\,(c_u u) - \Delta\left(\frac{c_r^2}{2}\right)\right\} \quad . \quad . \quad . \quad . \quad (28)$$

since $\Delta Q = 0$, and $\Delta L = 1/g\Delta\,(c_u u)$. For the present example, wherein $c_u = u$, and $c_r = c_{ro} = \text{const.}$, the application of Eq. 28 furnishes the new expression:—

$$\Delta i = \left(\frac{A}{g}\right)\left\{\Delta u^2 - \Delta\frac{u^2 + c_r^2}{2}\right\}\frac{A}{g}\Delta\frac{u^2}{2} \quad . \quad . \quad . \quad (29)$$

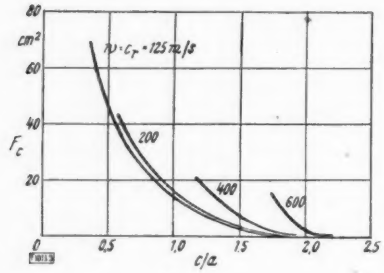


FIG. 3.

*Distribution of the cross-section $F_c$ dependent on $c/a$.*

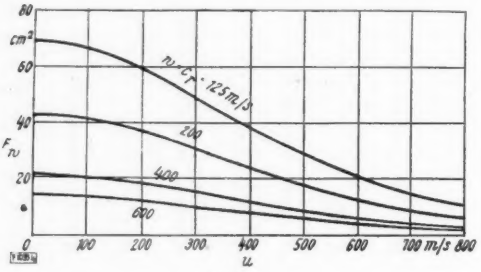


FIG. 4.

*Distribution of the cross-section $F_w$ (normal to the relative flow), dependent on the peripheral velocity. $u$.*

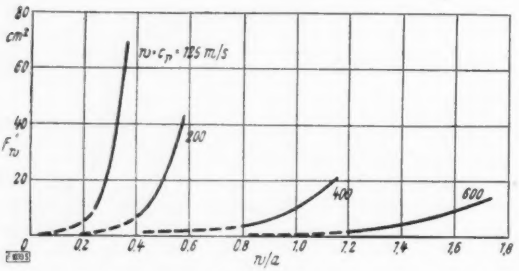


FIG. 5.

*Distribution of the cross-section $F_w$ dependent on $w/a$.*

The change in enthalpy with regard to some condition of reference, *e.g.*, the initial condition, can now be determined for any point on the rotor, and the corresponding peripheral velocity at that point. Thereafter, pressure and specific volume at any point of the wheel can be found by the *T-S* diagram, and the cross-section $F_c$ or $F_w$ at any point determined by applying the equation of continuity in the form:—

$$Gv = F_c c; \quad Gv = F_w w.$$

The curves of Figs. 2-5 were calculated by this means.*

* The initial condition has been specified as $u = 0$ in order to simplify calculation.

According to Eq. 29, the change of enthalpy is independent of $c_r$. This is explained by the fact that $c_r$ was assumed constant over the whole rotorm and thus does not play any part in the exchange of energy. The peripheral component of the absolute velocity naturally influences the change of enthalpy; this is merely not so obvious because it has been assumed that $c_u = u$, which has been introduced in this form into the equations.

Case " b ":—

$$c_u = u\,; \frac{dc_r}{du} = k. \quad \text{(Radial component uniformly accelerated or retarded.)}$$

Since $c_u = u$, the radial component of the absolute velocity is identical with the relative velocity, $dc_r/du = k$ therefore likewise indicates uniform acceleration or retardation of the relative velocity.

Examining the possibility of the occurrence of an extremum for the cross-section $F_c$ by applying Eq. 23, it is found that:—

$$\frac{a^2}{c^2} = 1 - \frac{2\frac{u}{c}}{\frac{c_r}{c}k + \frac{u}{c}} = \frac{\frac{c_r}{u}k - 1}{\frac{c_r}{u}k + 1}$$

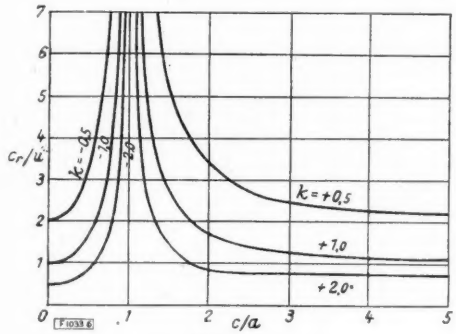


FIG. 6.

*Conditions for an extremum of the cross-section $F_c$ in relation to $c/a$.*

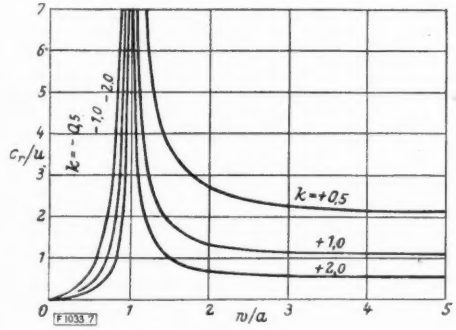


FIG. 7.

*Conditions for an extremum of the cross-section $F_w$ in relation to $w/a$.*

Similarly, Eq. 27 shows the occurrence of an extremum of the cross-section $F_w$ for the case:—

$$\frac{a^2}{w^2} = 1 - \frac{1}{\frac{c_r}{u}k} = \frac{\frac{c_r}{u}k - 1}{\frac{c_r}{u}k}$$

Figs. 6 and 7 show the connection which must exist according to the last two equations, between $c/a$ and $w/a$ respectively on the one hand, and $c_r/u$ or $k = dc_r/du$ on the other hand, if an extremum of the cross-sections $F_c$ or $F_w$ occurs. In the graphic representation, $k = dc_r/du$ has been applied as a parameter. Fig. 8 shows that an extremum of $F_c$ can occur only at high values of acceleration or retardation of the radial component of the velocity, say $1 < k < -1$, if the upper limit of the ratio $c_r/u$ is taken as unity. The possible extremum according to Fig. 8 for super- and sub-sonic velocities is a minimum, as will be clear on closer consideration.

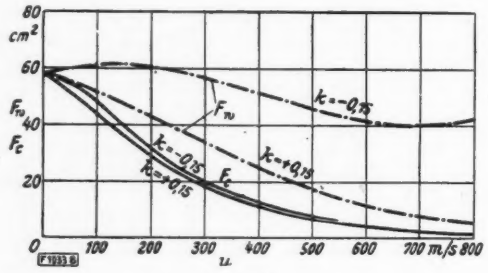


FIG. 8.

*Distribution of the cross-sections $F_c$ and $F_w$ in relation to the peripheral velocity $u$, for two specimen cases: $k = +0.15$ and $K = -0.15$.*

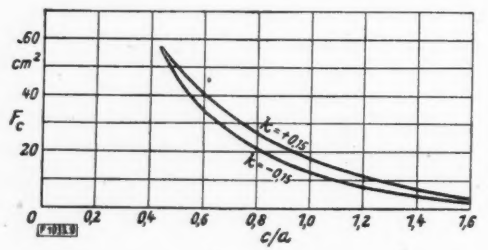


FIG. 9.

*Distribution of the cross-section $F_c$ in relation to $c/a$, for two specimen cases: $k = +0.15$ and $-0.15$.*

In so far as an extremum is psosible at supersonic velocity, the same conditions apply for the cross-section $F_w$, as for the cross-section $F_c$, at least in the qualitative sense. An extremum of $F_w$ at sub-sonic velocities, which according to Eq. 9 occurs at values of $k < 0$, is possible already at very small values of the ratio $c_r/u$, and small flow velocity. A closer examination will show that even two extremes are possible; a maximum can occur at large values of $w/a$, and a minimum is always present, viz., for small values of $w/a$. To conclude the

examination of Case " b," the development of the cross-sections has been plotted for $k=+0.15$, and $k=-0.15$, against possible practical values of $u$ (Fig. 8), and values of $c/a$ (Fig. 9), as the two practically valuable cases. The curves have been calculated exactly as in Case " a," excepting that in the determination of the change of enthalpy it has been necessary to consider the variation of the radial component of the absolute velocity thus:—

$$\Delta i=\left(\frac{A}{2g}\right)\left\{\Delta u^2-\Delta c_r^2\right\}$$

consequently, in the case of radial rotors with purely radial blades of normal design, the peripheral velocities such as are at present practicable, an extremum for the cross-section $F_c$ is impossible. A maximum for the cross-section $F_w$, however, can occur even at low speeds.

2. Radial Turbo-Rotors with Forward and Backward-Curving Blades.

The blade-forms will be represented in the established equations by imposing the condition $dc_u/du=\text{const.}=k$, where $k$ can be either positive or negative. Positive values of $dc_u/du$ will indicate increasing values of the peripheral component of absolute velocity with increasing peripheral velocity, *i.e.*, normal pump rotors with a flow radially outwards, or normal turbine rotors with a flow radially inwards. Negative values of $dc_u/u$ indicate the reverse cases, *i.e.*, pump rotors with the flow radially inwards, or turbine rotors with the flow radially outwards. The negative values therefore represent practically unimportant cases, and are included in the investigation solely for the sake of completeness. The value of $dc_u/du=\pm co,(?)$ which has likewise been included in the investigation, corresponds to a rotor wherein the deflection of the flow by the blades takes place at constant peripheral velocity. To avoid multiplying the possible variants unnecessarily, the radial component of the flow velocity will be assumed invariable, *i.e.*, $dw_r/du=dc_r/du=0$. Within the range of practical interest, the influence of variation of the radial component of flow velocity will be inconsiderable, since a small variation in the velocity will be accompanied by a correspondingly small variation in the thermodynamic state, *i.e.*, the pressure and the temperature. By Eq. 23, accordingly, an extremum of the cross-section $F_c$ will occur when:—

$$\frac{a^2}{c^2}=1-\frac{\left(k+\frac{c_u}{u}\right)}{\left(\frac{c_u}{u}k\right)}=1-\frac{1}{\left(\frac{c_u}{u}\right)}-\frac{1}{k}$$

Similarly, by Eq. 27, an extremum for the cross-section $F_w$, when:—

$$\frac{a^2}{w^2}=1-\left\{\frac{1}{\left(1-\frac{c_u}{u}\right)1-k)}\right\}$$

The last two equations are reproduced in the curves of Figs. 10 and 11, with $c_u/u$ as ordinates, $c/a$ and $w/a$ respectively, as abscissæ. The value of the ratio $c_u/u$ gives an indication whether the blades are forward curving or backward curving. The upper limit for backward curving blades can be assumed as $(c_u/u)_p\approx 0.8$—the expression $(c_u/u)_p$ representing the values of the ratio $c_u/u$ on the pressure side of a pump or turbine. This limit further depends, however, on the ratio of the radii and the angle of entry of the blade in pumps, or the angle of outlet of the blade in turbines. At $(c_u/u)_p>1$, the blades are hook shaped.

By means of the curves in Figs. 10 and 11 it will now be possible to examine whether an extremum of the cross-section can occur for compressible fluids in radial rotors of usual design with normal blade forms.

The upper limits of normal design for turbo-compressors can be taken as: $c_u/u=1$, and $dc_u/du=k=2$. The lower limits can be taken as $c_u/u=0$, and

$dc_u/du = k = 0$. This implies that, *e.g.*, if the ratio of outlet to intake diameter of the rotor is 2, and assuming an entering flow free from twist, the ratio $c_u/u$ at the outlet of the rotor will not exceed unity. According to Figs. 10 and 11, an extremum will not occur within these limits either for the cross-section $F_c$ or for the cross-section $F_w$. When the value of $dc_u/du = k$ begins to exceed unity, an extremum for $F_w$ quickly occurs.

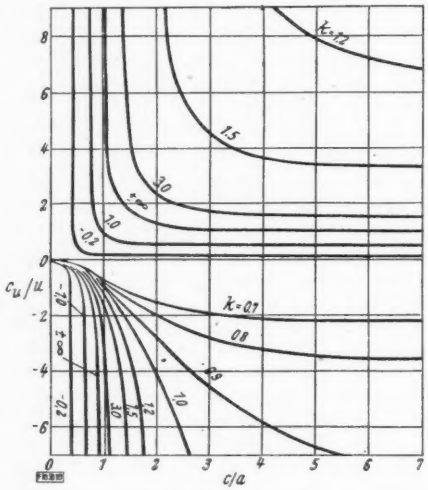


FIG. 10.

*Conditions for an extremum of the cross-section $F_c$ in relation to $c/a$, for the case $dc_u/du = k$ ($dc_r/du = dw_r/du = 0$).*

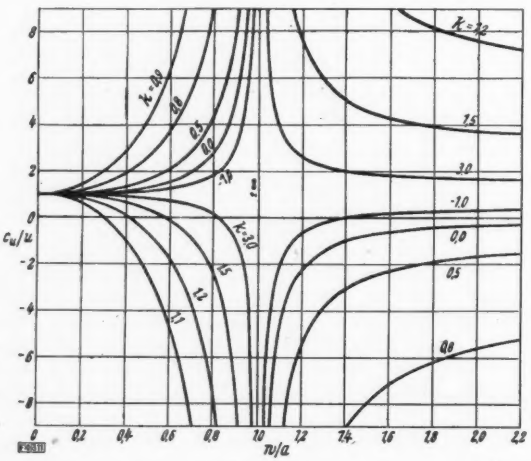


FIG. 11.

*Conditions for an extremum of the cross-section $F_w$ in relation to $w/a$, for the case $dc_u/du = k$ ($dc_r/du = dw_r/du = 0$).*

Other types of pump rotors, *e.g.*, axial rotors, will not be considered in this section.

For turbines with a flow in radial planes, $dc_u/du=k$ can assume both positive and negative values, since the flow can be either radially inwards or radially outwards (Ljungstrom turbine, Elektra turbine). In this case the absolute values of $k$ are usually very great, since the difference of peripheral velocity between the intake and outlet of a blade is very small, while the variation of the peripheral component of absolute velocity is very great. Values of $dc_u/du=\pm 10$, and $c_u/u=\pm 2$ to 3, are not uncommon. In accordance with Fig. 10, the cross-section

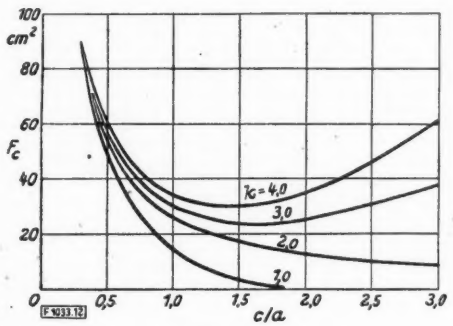


FIG. 12.

*Distribution of the cross-section $F_c$ in relation to $c/a$ for some examples ($k=1$ to 4) of the case $dc_u/du=k$; $dc_r/du=0$.*

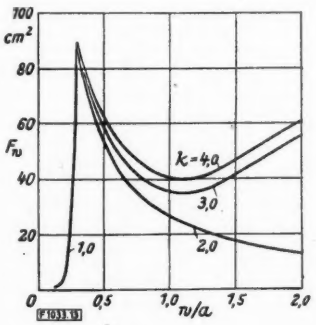


FIG. 13.

*Distribution of the cross-section $F_w$ in relation to $w/a$ for some examples ($k=1$ to 4) of the case $dc_u/du=k$; $dc_r/du=dw_r/du=0$.*

$F_c$ can in such case possess an extremum for any value of the ratio $c/a$. Similarly, Fig. 11 shows that in principle the cross-section $F_w$ can have an extremum for any ratio of $w/a$. For such high positive and negative values of $dc_u/u$, the ratio $w/a$ in the majority of cases very closely approaches unity. This is because the difference of peripheral velocity between inlet and outlet edge of a blade in radial-flow turbines is very small and the difference between these and axial-flow turbines, in which an extremum of the cross-section $F_w$ is always present at $w/a=1$, is thus very slight. This can also be seen from Fig. 11, since $dc_u/du=k=\pm\infty$ corresponds to the conditions for an axial turbine.

The development of $F_c$ and $F_w$ respectively against $c/a$ and $w/a$, has been numerically determined for a number of values of $dc_u/du=k$, and reproduced in Figs. 12 and 13. The calculation is based on an initial condition corresponding to $p_o=1$ atm. abs., $t_o=20°C.$, $u_o=0$, $c_{ro}=c_r=100$ m./s. $=$const. and $c_{uo}=0$. The basic values $a=\sqrt{dp/d\rho}$ and $v$ for the rotor were determined in the same manner as previously described. With the assumptions already indicated, which are permissible for the case of a turbo-compressor rotor, $dc_u/du=2$ corresponds to an impulse rotor. For $dc_u/du=k<2$, the pressure at the rotor outlet is greater than the entry pressure; for $dc_u/du=k>2$, despite the influx of energy from the rotor, the outlet pressure is less than the intake pressure. A minimum cross-section, both for $F_c$ and $F_w$, is only possible in the second case, *i.e.*, when $k>2$. This is clearly apparent from Figs. 10 to 13.

Owing to the multiplicity of boundary conditions, a numerical calculation of the cross-sectional distribution of a radial steam turbine has not been attempted. The general examination of the problem must suffice.

### 3. Axial-Flow Centrifugal Rotors.

As is well known the exchange of energy in axial rotors proceeds at constant peripheral velocity. The axial rotors are therefore sufficiently defined by the condition $u=$const. This, of course, says nothing as yet, regarding the form of the blades. However, as will be seen later, a definition of the blade-form is unnecessary in this case, if examination is confined to an extremum of the cross-section $F_w$. This aspect of the problem can be suitably investigated by the application of Eq. 28.

Omitting the trivial case that $dw$ and $du$ are simultaneously equal to zero, it will be found that an extremum, viz., a minimum of the cross-section $F_w$, is required for any axial rotor, if the relative velocity equals the sonic velocity $a=dp/d\rho$ for the particular state. The explanation is very simple: Since the peripheral velocity is constant, no variation in pressure can be produced by centrifugal forces. The pressure-variation can therefore be produced only by variation of the relative velocity. The latter is consequently related to the pressure in the same manner as the absolute velocity in a stationary channel.

The requirement of a minimum cross-section at $w/a=1$ for axial-flow rotors, is already known in the design of steam turbines.

By Eq. 12, an extremum of the cross-section $F_c$ is necessary when:—

$$\frac{a^2}{c^2}=1-\frac{d\,(c_u u)}{(c\,dc)}=\frac{\{\,c\,dc-d\,(c_u u)\,\}}{(c\,dc)}$$

On account of Eqs. 13 and 16, this can also be written as:—

$$\frac{a^2}{c^2}=\frac{(w\,dw)}{(c\,dc)}=\frac{dw^2}{dc^2} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

since for axial rotors $du=0$. To determine whether an extremum of the cross-section $F_c$ is necessary, a knowledge of $dw^2/dc^2$ is still required. Since $a^2/c^2$ can only be positive, it is necessary to postulate that the relative and the absolute velocities increase or decrease simultaneously. As closer inspection will show, the extremum then possible, will always be a minimum.

It will now be briefly investigated, whether a minimum for the cross-section $F_c$ is necessary in the usual designs of axial-flow rotors for compressible fluids. This investigation can be limited for steam turbines, to the consideration of reaction rotors, since impulse rotors will be treated separately. The usual halving of the degree of reaction in reaction turbines, produces equal blade-velocity diagrams for intake and outlet, as shown by Fig. 14. For this case, therefore, $dw^2/dc^2$ will always be negative, and a minimum for the cross-section $F$ is possible. Obviously, blade forms are possible at which $dw^2/dc^2$ is positive, but these are of no practical interest.

Axial blowers may be regarded as a reversal of the case of the reaction turbine, particularly if they are designed for low delivery volumes, but high delivery

heads. It is likewise possible in this case to take advantage of the method of halving the reaction, by using the same blade form for rotor and guide-ring. The resulting blade-velocity diagrams are approximately those shown in Fig. 15, $dw^2/dc^2$ is, of course, negative in this case also.

Finally, the most usual type of single-stage axial blower, the fan type, will be examined. In this case, assuming the flow to the wheel to be free from twist, the appearance of the blade-pressure diagrams will be similar to Fig. 16. Again, $dw^2/dc^2$ is negative. It will be seen, therefore, that an extremum of the cross-section $F_e$ is unnecessary for the usual designs of axial rotors, even if the peripheral velocity is further increased.

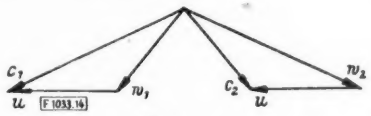


FIG. 14.

*Velocity diagrams of axial reaction turbines.*

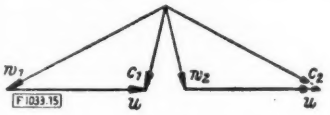


FIG. 15.

*Velocity diagrams of axial reaction turbines with halved reaction.*

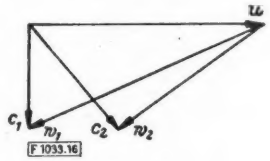


FIG. 16.

*Velocity diagrams of impeller fans.*

4. IMPULSE ROTORS.

In rotors of the impulse type, the exchange of energy proceeds at constant pressure from the inlet to the outlet of the rotor. Thus, disregarding losses, the exchange also proceeds at constant specific volume. Consequently, at uniformly increasing or decreasing velocities, an extremum of the cross-section cannot occur, as will also be seen from Eqs. 14 and 17.

## IV. THE POSSIBILITY OF THE OCCURRENCE OF COMPRESSION SHOCKS IN TURBO-ROTORS.

An investigation into this problem is of interest for the engineer, even more by reason of the consequent energy losses than on account of the problem of the extremum of the cross-section. The possibility of the occurrence of compression shocks will be shown here on diagrams first constructed by *Busemann** for flow types without influx of mechanical energy, suitably modified for the representation of a flow with influx of mechanical energy.

If $c$ be selected as the abscissa of this state-curve (Fig. 17), the ordinate will be given by:—

* A. Busemann. "Gasdynamik, Handbuch der Exp.—Physik" (Gas Dynamics, Handbook of Experimental Physics), Vol. IV, 1, Leipzig 1931.

$$\int_{p=o}^{p=p} \{ dp - \rho d (c_u u) \}$$

as will be clear from subsequent explanations. There is no restriction in principle, on the selection of the lower limit of the integral which can most suitably be taken as $p-o$. The connection between $c$ and

$$\int_{p=o}^{p=p} \{ dp - \rho d (c_u u) \}$$

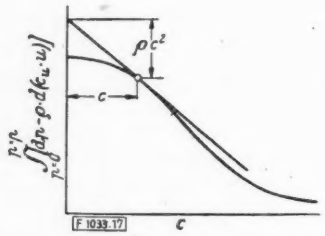


FIG. 17.

*Compression shock in a flow with absorption or abstraction of energy.*

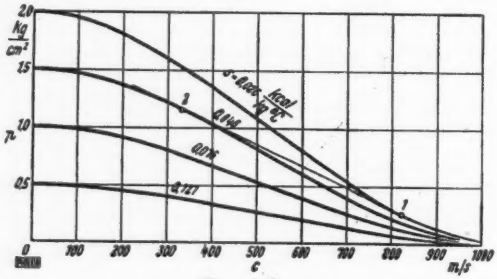


FIG. 18.

*Adiabatics of the* $\int \{ dp - \rho d (c_u u) \}$, $c$ *diagram with energy absorption according to the equation*

$$\frac{d (c_u u)}{d (c^2/2)} = 0.5$$

*In this particular case of energy absorption, the integral can be replaced as ordinate by the value* $\{ 1/(1-k) \}\, p$, *or simply* $p$, *in accordance with Eq.* 31.

can be calculated from the equation of energy, *e.g.*, by assuming an adiabatic state change. As an example, Fig. 18 shows a series of such adiabatics in the state diagram. These adiabatics were calculated on the assumption that the transmission of energy from the rotor to the flow proceeds under the condition that :—

$$\frac{d (c_u u)}{d (c^2/2)} = \text{const.} = k$$

since for this case the curve of $\int \{ dp - \rho d (c_u u) \}$ can be quite simply calculated, viz. :—

$$\int_o^p \{ dp - \rho d (c_u u) \} = \left\{ \frac{1}{(1-k)} \right\} \int_o^p dp = \left\{ \frac{1}{(1-k)} \right\} p \quad . \quad . \quad (31)$$

In this special case, therefore, the integral can be substituted by $\{1/(1-k)\}p$, or even along the ordinate $p$, as done in Fig. 18. For the calculation of the curves in this figure, it was assumed that:—

$$\frac{d(c_u u)}{d(c^2/2)} = k = 0.5$$

since by Fig. 1 this value of $k$ corresponds with certainty to the requirement of an extremum for the cross-section. The pressure $p$ was determined by Eq. 28 which in this special case can be written in the form:—

$$\Delta i = \frac{A}{g}(k-1)\Delta\frac{c^2}{2}$$

The value of $\Delta i$ can be calculated for any values of $c$, or $\Delta c^2$, whence, by assuming an initial condition, the ultimate condition and thus the pressure, are obtainable by an $i-S$ or $T-S$ diagram. The present diagram was calculated for air, assuming an initial heat content of $i_0 = 64.2$ kcal./kg., corresponding to a temperature $t_0 = 100°C$. The adiabatics were calculated for a number of initial pressures.

The conditions for the occurrence of compression shocks can now be determined by application of the equation of energy, the equation of continuity, the second thermodynamic law, and the impulse theorem. Whereas the first three conditions must be satisfied in any state change, in the present case, since a shock phenomenon is involved, the impulse theorem must also be applied. The energy theorem is satisfied automatically for any point of the curve, by the fact that the calculation has already been effected by use of that theorem. Application of the equation of continuity creates a special condition; the cross-section remains constant during the compression shock, whence it follows that:—

$$c\rho = \frac{M}{F} = \frac{G}{gF}$$

must have the same value before and after the compression shock. $c\rho$ can be termed the intensity of the flow. As the following calculation shows, the negative direction of the tangent to the adiabatics of the $\int\{dp - \rho d(c_u u)\}$, $c$ curve, will represent the flow intensity $c\rho$, since:—

$$-\frac{d\int\{dp-\rho d(c_u u)\}}{dc} = -\left\{\frac{dp}{dc} - \rho\frac{d(c_u u)}{dc}\right\} = \rho c \quad . \quad . \quad (32)$$

Thus, the tangents to the adiabatics at points corresponding to states before and after the compression shock must have the same direction. Application of the impulse theorem imposes a further condition, which, together with the above-mentioned, defines the state points before and after the compression shock. But the impulse theorem of flow mechanics, applied to a centrifugal rotor, is none other than the previous Eq. 13.

Remembering this, and that during the development of the shock the cross-section remains constant—and thus $c\rho = M/F$ must also be constant—the conditions before and after the shock are defined by:—

$$\int_1^2 \{dp - \rho d(c_u u)\} + \overset{2}{\underset{1}{\Delta}}(c^2\rho) = 0 \quad . \quad . \quad . \quad . \quad (33)$$

The index 1 applies to the state before the shock, and index 2 to the state after the shock. Fig. 17, however, shows that the tangents to the adiabatics cut off on the ordinate the distance:—

$$\int_{p=0}^{p=p} \{dp - \rho d(c_u u)\} + c^2\rho$$

Consequently, this distance remains constant during the development of the shock. From this, combined with the first condition resulting from application

of the equation of continuity, it follows that the state points before and after the shock must have the same tangent. The points 1 and 2 in Fig. 18, for instance, are such points. The direction of the state change is determined by the second law of thermodynamics, *e.g.*, the state change between points 1 and 2 of Fig. 18, can only proceed in the direction from 1 to 2, since only in that direction will the entropy increase.

The adiabatics in the $\int \{ dp - \rho d (c_u u) \}$, $c$ diagram, all have a point of inflection, corresponding to a minimum of the cross-section, since at this point the inclination of the tangent, and consequently the flow intensity $c\rho$, attain the maximum. Compression shocks are therefore only possible if the conditions of state change from points on the right to points on the left of the inflection. The point of inflection, *i.e.*, the condition requiring a minimum of the cross-section, is thus the limiting point for the occurrence of compression shocks, and in this respect the behaviour of a flow with influx of mechanical energy is the same as of a flow with only a conversion of energy. The surprising fact emerges, however, that compression shocks are possible even when the absolute velocity $c$, is less than the velocity of sound, $a = \sqrt{(dp/d\rho)}$. It is true that for the example chosen, with $d (c_u u)/d (c^2/2) = k = 0.5$, the minimum for the cross-section was found at $c/a = 1.41$, but the curves of Fig. 18 could equally well be plotted for other values of $k$, or even for cases wherein the value of $d (c_u u)/d (c^2/2)$ is variable over the flow path through the rotor. All types of rotor could be dealt with by this means, as has already been done in the preceding chapters, although the basic equations were different. These preceding investigations frequently led to a minimum cross-section $c/a < 1$. It remains to be explained, however, how compression shocks can arise at values of the flow velocity $c$, less than the velocity of sound $a = \sqrt{(dp/d\rho)}$.

This apparent contradiction is explained by the fact that in a flow with addition or abstraction of energy the velocity

$$a = \sqrt{(dp/d\rho)}$$

no longer corresponds to the velocity of propagation of small disturbances. This method will be considered further in the following section while retaining the term " sonic " velocity for the velocity under these conditions.

## V. Derivation of the Sonic Velocity in a Flow with Influx or Efflux of Energy.

The investigation follows in exactly the same manner as the derivation of the sonic velocity in a stationary medium of uniform density.* It starts with Euler's equation of absolute motion, modified for the effect of the addition of energy :—

$$\frac{\partial c}{\partial t} + c\frac{\partial c}{\partial s} = \frac{\partial (c_u u)}{\partial s} - \left(\frac{1}{\rho}\right)\frac{\partial p}{\partial s} \quad . \quad . \quad . \quad . \quad (34)$$

where $\partial (c_u u)/\partial s$ is the force per unit mass relating to the influx of energy. If all flow velocities and fluctuations of pressure and density be assumed small, the term $c\partial c/\partial s$ can be omitted in the first instance. The density can be expressed as :—

$$\rho = \rho_0 (1 + \sigma) \quad . \quad . \quad . \quad . \quad . \quad (35)$$

wherein $\sigma$ is a small quantity. Furthermore, both $p$ and $c_u u$ will be considered as functions of the density alone. Hence, by Eq. 34 :—

$$\frac{\partial c}{\partial t} = \left(\frac{1}{\rho}\right)\left\{\frac{\rho d (c_u u)}{d\rho} - \frac{dp}{d\rho}\right\}\left(\frac{\partial \rho}{\partial s}\right) \quad . \quad . \quad . \quad (36)$$

By Eq. 35 :—

$$\frac{\partial \rho}{\partial s} = \rho_0 \frac{\partial \sigma}{\partial s} \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

* H. Lamb. " Lehrbuch der Hydrodynamik " (" Handbook of Hydrodynamics "). 2nd Ed., Leipzig, Berlin 1931.

and putting

$$\left\{\frac{[dp-\rho d\,(c_u u)]}{d\rho}\right\}_{\rho=\rho_0}=(a^*)^2 \quad . \quad . \quad . \quad . \quad (38)$$

the following equation is obtained :—

$$\frac{\partial c}{\partial t}+(a^*)^2\frac{\partial \sigma}{\partial s}=0 \quad . \quad . \quad . \quad . \quad . \quad (39)$$

From the equation of continuity :—

$$\frac{\partial \rho}{\partial t}+\frac{\partial\,(\rho c)}{\partial s}=0 \quad . \quad . \quad . \quad . \quad . \quad (40)$$

it follows by the assumptions made, that :—

$$\frac{\partial c}{\partial s}+\frac{\partial \sigma}{\partial t}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

Eliminating $c$ from Eqs. 39 and 41 :—

$$\frac{\partial^2 \sigma}{\partial t^2}=(a^*)^2\frac{\partial^2 \sigma}{\partial s^2}$$

hence :—

$$a^*=\sqrt{\{[dp-\rho d\,(c_u u)]/d\rho\}}$$

is the velocity of propagation of small disturbances (" sonic " velocity) in the presence of addition or abstraction of energy.

## VI. Introduction of the " Sonic " Velocity $a^*$ into the Preceding Equations.

In Chaps. I-IV, the reference velocity was throughout the velocity of sound by the formula: $a=\sqrt{(dp/d\rho)}$. This, however, did not provide results of sufficient clarity. Consequently, it will now be replaced by the " sonic " velocity $a^*=\sqrt{\{[dp-\rho d\,(c_u u)]/d\rho\}}$ according to Eq. 42.

The most general form of equation for an extremum of the cross-section was given at the beginning by Eq. 3 :—

$$\frac{c}{v}=\frac{dc}{dv}=\frac{c\,dc}{c\,dv}$$

which can also be written, by taking Eq. 13 into consideration, as :—

$$c^2=\frac{[v d\,(c_u u)-dp/\rho]}{dv} \quad . \quad . \quad . \quad . \quad . \quad (42)$$

or

$$c^2=\left(\frac{v}{\rho}\right)\left(\frac{[\rho d\,(c_u u)-dp]}{dv}\right) \quad . \quad . \quad . \quad . \quad (43)$$

Since :—

$$v=\frac{1}{\rho g} \quad \text{and} \quad \frac{dv}{d\rho}=-\left(\frac{1}{g}\right)\cdot\left(\frac{1}{\rho^2}\right)$$

it follows from Eq. 43, that :—

$$c^2=\frac{dp-\rho d\,(c_u u)}{d\rho}=(a^*)^2$$

This clearly proves that an extremum of the cross-section will always occur, even with an influx or efflux of mechanical energy, when the flow velocity is the same as the sonic velocity of the corresponding state; in the case of an external exchange of energy, the sonic velocity must equal :—

$$a^*=\sqrt{\{[dp-\rho d\,(c_u u)]/d\rho\}}$$

Similarly, compression shocks are only possible if the absolute flow velocity $c$ is greater than the sonic velocity $a^*$, since Chap. IV shows that compression

* H. Lamb. " Lehrbuch der Hydrodynamik " (" Handbook of Hydrodynamics "). 2nd Ed., Leipzig, Berlin 1931.

shocks can only occur if the flow velocity $c$ is greater than the flow velocity corresponding to the least cross-section. This resolves all previous uncertainties, and further indicates that the absolute velocity in a rotor is decisive for the hydrodynamic behaviour of the flow, since similar connections cannot be established for the relative velocity and remembering that the analysis of compression shocks also requires utilisation of the absolute velocity. These rather significant results thus easily fall into line with our experience on the behaviour of supersonic flow without the exchange of mechanical energy.

For the case when the exchange of energy between the blades of the rotor and the gas flow can be represented by the expression

$$\frac{d\,(c_u u)}{d\,(c^2/2)} = \text{const.}$$

the sonic velocity $a^*$ can easily be calculated.

By Eq. 13, the square of the sonic velocity $a^*$ can be written as:—

$$(a^*)^2 = \frac{\{\, dp - \rho d\,(c_u u)\,\}}{d\rho} = -\frac{c\rho dc}{d\rho} \quad . \quad . \quad . \quad . \quad (44)$$

The term

$$\frac{d\,(c_u u)}{d\,(c^2/2)} = \text{const. } k$$

will now be introduced into the preceding equation.

By Eq. 13:—

$$\frac{d\,(co_u u)}{cdc} = \left(\frac{1}{\rho c}\right)\cdot\left(\frac{dp}{dc}\right) + 1 = k\,; \text{ or } c\rho\left(\frac{dc}{dp}\right) = \frac{1}{(k-1)}$$

which can also be written as:—

$$c\rho\left(\frac{dc}{d_f}\right)\left(\frac{d\rho}{dp}\right) = \frac{1}{(k-1)}$$

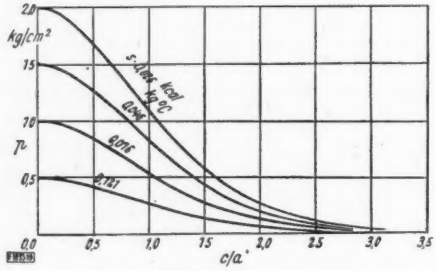


FIG. 19.

*Adiabatics of the* $\int \{\, dp - \rho d\,(c_u u)\,\}$, $c/a^*$ *diagram with energy absorption according to the equation*

$$\frac{d\,(c_u u)}{d\,(c^2/2)} = 0.5$$

*According to Eq. 31 the integral can be replaced as ordinate by the value* $\{\, 1/(1-k)\,\}\, p$, *or simply* $p$.

Introducing the sonic velocity $a = \sqrt{(dp/d\rho)}$ the expression becomes:—

$$\frac{dc}{d\rho} = \frac{a^2}{(k-1)}$$

* H. Lamb. "Lehrbuch der Hydrodynamik" ("Handbook of Hydrodynamics"), 2nd Ed., Leipzig, Berlin 1931.

Comparing with Eq. 44, the expression for the sonic velocity is finally obtained as :—

$$(a^*)^2 = \frac{a^2}{(1-k)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

This simple expression is, however, only applicable to the particular case considered. The equation has been used in Fig. 19 to recalculate the curves of Fig. 18 for the abscissa $c/a^*$.

## VII. Conclusions on the Design Features of Centrifugal Machinery for Supersonic Velocities.

It will now be attempted—to the extent possible by merely theoretical consideration—to outline the principles of construction of centrifugal machinery operating at supersonic velocities, making use of the results obtained in the present investigation. The guiding intention will be, to avoid as far as possible, the formation of sonic waves, and practical elimination of compression shocks, in order to obtain a minimum of the so-called wave-making resistance.

To begin with, therefore, it must be borne in mind that the critical flow velocity—*i.e.*, the sonic velocity—is different in the stationary components of the machine, *e.g.*, guide-vanes and volutes, and in the moving parts, *e.g.*, the rotor. For a numerical evaluation of flow conditions, it will therefore be advantageous to select as the velocity of reference for the flow conditions in the rotor, the sonic velocity " $a$ " corresponding to the stationary parts (stator assembly), since it is then easiest to determine for any given blade-form, whether the flow conditions in the stator or the rotor are the most critical. The investigations made in Chap. III have shown that in radial and axial rotors of usual design, there is either no critical flow velocity whatever, or this velocity is greater than the sonic velocity " $a$ "; in other words, the flow in the stator assembly of a centrifugal machine is the most exposed to critical conditions, and requires the most attention in designing. On the other hand, the preceding calculations only indicate whether compression shocks are possible, not whether they actually occur. The occurrence of compression shocks, under appropriate conditions, depends on the presence of sources of disturbance. The nature of the possible disturbances in a centrifugal machine, is difficult to predict. In any case, every obstacle, and every curved deflecting surface, form such a source. However, the unsteady flow in the rotor, consequent on the presence of a finite number of blades, and the irregular distribution of the peripheral velocity in the rotor, particularly at the outlet, are further causes of compression shocks. The actual extent of the effect of these factors can of course only be determined by experiment.* In any case, it must be endeavoured to avoid all obstacles in the flow. Obstacles in this sense, are the blades, particularly the leading edges, and all irregularities in the boundary walls. This last source of disturbance can be considerably diminished by suitable mechanical treatment of the walls, and disturbances from this cause may be considered avoidable. The elimination of disturbance owing to the finite thickness of the blades, is more difficult. Firstly, the harmful influence of the leading edge of the blade, should be controlled by proper tapering of the tips and roots. So long as the tangent to the blade at the tip coincides in direction with the relative velocity at the leading edge of the blade, thus producing a " shock-free " entry, tapering should be of considerable benefit. When, however, these directions no longer coincide owing to alteration of the delivery head, tapering does not improve the entry conditions; on the contrary, in such case deflection of the flow in the direction of the blade tangent by means of an oblique compression shock, will assuredly take place. Tapering the blades therefore affords a remedy only in one particular running condition, and the question of eliminating the disturbing influence of the blades, remains unsolved.

* The extent to which the derived relations are modified by a finite number of blades, will not be further dealt with.

Since, however, as already stated, compression shocks in the rotor need hardly be feared, the question applies only to the stator blades and the leading edges of the rotor blades. Even though in accordance with the results of the present investigation, no sonic waves, and thus no compression shocks, can arise inside the rotor, such sonic waves and compression shocks can of course arise at the leading edges of the rotor blades, if the relative entry velocity exceeds the sonic velocity $a$. Probably, however, this wave formation does not extend far into the rotor blade channel—at least judging by the results of the present investigation. Nevertheless, it must still be borne in mind that these results apply strictly only for an infinite number of blades, and great care should be exercised when applying them to rotors with a finite number of blades.

Usually, however, in the case of radial rotors the relative velocity at the rotor entry will be less than the sonic velocity. In axial rotors, the case is different; here, at high peripheral velocities and freedom from twist in the inflow, the relative entry velocity will in many cases exceed the sonic velocity. In the majority of cases, however, the relative velocity at the rotor entry can be reduced below sonic velocity, by giving the absolute flow a peripheral twist before it enters the rotor, *i.e.*, by providing guide vanes in advance of the rotor. The resulting velocity diagram for the entry is shown on Fig. 20*a*; the corresponding entry velocity diagram in the absence of twist, is shown on Fig. 20*b*.

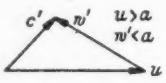


FIG. 20*a*.

*Reduction of the relative inflow velocity to the rotor at subsonic velocity by an absolute flow with twist in the direction of the peripheral velocity.*

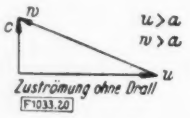


FIG. 20*b*.

*Diagram of the entry velocity for inflow without twist.*
*Zuströmung = inflow.*
*mit-ohne Drall = with-without twist.*

The problem therefore remains of eliminating the heavy losses in the guide vanes, accompanying the conversion of supersonic velocity into pressure. The simplest remedy, is to abolish the guide vanes altogether; if this is impossible or undesirable for other reasons, the flow can be decelerated to subsonic velocities, after leaving the rotor, in a smoothly-finished guide-ring, before being led through the guide vanes of the diffusor. Obviously, the cross-section $F_c$ should attain a minimum when sonic velocity is reached. This does not, however, imply that the width of the guide-ring must necessarily be reduced to minimum, since the cross-section between two streamlines can attain the necessary minimum as soon as the angle between the streamlines and the periphery, assumes a minimum value. The supplementary losses by wave-formation in the guide vanes can of course also be eliminated by appropriate design of the rotor blades, making the absolute discharge velocity from the rotor less than the sonic velocity $a$, and reducing the consequent losses in delivery head by giving the flow a negative twist before entering the rotor. This method, however, is of little advantage, since besides the loss in delivery head, the considerable retardation in the

relative velocity causes increased losses at the blades. In addition, the blade form resulting by this method is unsuitable from considerations of strength, for the high peripheral velocities encountered.

Up to now, the difficulties occasioned by the considerable velocity gradient across the blade channel have only been briefly indicated. They consist firstly in the circumstance that this velocity distribution causes the sonic velocity to be easily attained or exceeded locally, while the mean velocity remains considerably below that value. This may primarily cause local compression shocks; on the other hand, it must be expected that the irregular flow conditions at the rotor outlet, producing partly supersonic and partly subsonic velocities, will give rise to considerable difficulties. Very possibly, therefore, it will be found advantageous to increase the number of blades in centrifugal machines intended to deal with working media flowing at supersonic velocity, in order to reduce, at least in some degree, the magnitude of the velocity fluctuations over the periphery of the rotor. All these problems still require thorough experimental investigation, particularly in respect to the influence of a finite number of blades.

## VII. Summary.

The question dealt with is, whether an extremum of the cross-section is required for the case of a flow through a turbo-rotor (*i.e.*, a flow wherein an exchange of energy with the surrounding medium takes place), when the flow velocity exceeds the velocity of sound. Equations of general application are derived, proving that both for infrasonic and for ultrasonic (supersonic) velocities, an extremum of the cross-section is possible, " sonic " velocity being defined as $a=\sqrt{(dp/d\rho)}$. This applies equally to the cross-section $F_w$ normal to the lines of relative flow, and to the cross-section $F_c$, normal to the lines of absolute flow. These general equations are then applied to some special cases of rotor and blade forms. The result is obtained that (with limitation to the forms of rotors and blades usual in centrifugal machinery) an extremum of the cross-section $F_c$ can occur only in radial steam turbines. An extremum of the cross-section $F_w$, on the other hand, is more frequent. Thus, a minimum is found for the cases of radial steam turbines and axial rotors; in the latter case invariably for $w/a=1$. Furthermore, a maximum of the cross-section $F_w$ is found in radial rotors with considerable radial differences, *e.g.*, turbo-compressor rotors.

The conditions for the occurrence of compression shocks in centrifugal rotors are next investigated. It was found that the same conditions are applicable, as were derived for the extremum of the cross-section $F_c$. The fact that an extremum of the cross-section $F_c$, and therefore also compression shocks, can occur at any value of the ratio $c/a$, gave occasion to replace

$$a=dp/d\rho \text{ by } a^*=\sqrt{\{[dp-\rho d(c_u u)]/d\rho\}}$$

as the expression for the velocity of reference; $a^*$ being derived as the velocity of sound (sonic velocity) in a gas flow absorbing or emitting a quantity of energy $L=(1/g)\,d(c_u u)$.

Thus, within the sphere of influence of the rotor, the sonic velocity of the working medium in a centrifugal machine is defined by the equation:—

$$a^*=\sqrt{\{[dp-\rho d(c_u u)]/d\rho\}}$$

Introducing $a^*$ as the reference velocity, the condition for an extremum of the cross-section $F_c$ and lower limit for the occurrence of compression shocks is obtained for the most general case as: $a^*/c=1$. This result links up easily with the general laws applying to gases, in the absence of a state-change caused by the exchange of mechanical energy with the surrounding medium; as *e.g.*, in discharge nozzles. These laws are, of course, included in the above result as a special case. Finally it has been attempted with the help of the results obtained, to define some requirements for the suitable design of centrifugal machines operating at supersonic velocities, from the guiding aspect of the avoidance or diminution of wave-formation and compression shocks.

## REVIEWS.

TEACH YOURSELF AIR NAVIGATION.

By " Kaspar." English Universities Press. 1942. 2/6.

Here is yet another of that spate of books by which the fortunate air-minded youth of to-day is encouraged to " teach himself " more than his elders have ever forgotten—and which prove a blessing or a curse to his future instructors, accordingly as to whether or not he realises that there still remains something that his elders can teach *him*.

Like its companions " Teach Yourself Trigonometry " and " Teach Yourself Mathematics," this book is obviously written by a man with a good grasp of his subject and a facility for imparting it.

The subject-matter adheres fairly closely to the pre-war syllabus for the 2nd Class Civil Navigator's Licence; this probably accounts for the forcible insertion of a 16-page chapter on Meteorology, which over compression has made somewhat confusing. The rest of the book is, within its limits, lucid and satisfying, and fulfils its avowed purpose of imparting the fundamental principles of Air Navigation in a form that can be understood by students who know no trigonometry and next to no mathematics.

DIRECTION-FINDING HANDBOOK FOR WIRELESS OPERATORS.

By W. E. Crook. Sir Isaac Pitman and Sons. 1942. 3/6.

When reviewing a previous manual in this excellent series for the wireless operator, we suggested that the notes on Direction-Finding might well be amplified in a subsequent edition. The author has done better; he has written a complete book. In his succinct and practical style he first deals with the subject in general, and then with its main applications: the Rotating Frame and the Bellini-Tosi systems. His aim, as he says in the preface, is not to describe the working of any particular set, which will inevitably become obsolete, but to give his readers a grasp of general principles which they themselves can apply to the apparatus they have to operate. A very useful chapter deals with Sensing and Fixing—aspects which are frequently passed over with only a brief mention—and all sources of error are dealt with exhaustively.

The level of technical knowledge assumed in the reader is somewhat higher than in the preceding manuals, but should present no difficulties to those who have read them.

ASTROGRAPHICS: OR FIRST STEPS IN NAVIGATION BY THE STARS. (A PRIMER FOR THE AIRMAN CADET.)

By Frank Debenham. W. Heffer and Sons. 1942. 7/6.

When a book on a subject as reputedly abstruse as Astro-Navigation begins with a quotation from " The Hunting of the Snark " and continues with the rhyme:—

A tiny but precocious babe
Was playing with an astrolabe;
Her older brother, not so frail,
Was toying with a stereo-scale,

the reader is entitled to expect that it will be good fun. Such good fun is this particular book that the reviewer, taking it to bed from a sense of duty, found himself still wide awake and enjoying it at 2 a.m.

Here we learn of the missionary and the trader who lived close to the date line: " By suitable journeys the missionary contrived to have 104 Sundays in the year and the trader, contrariwise, contrived to have no Sundays at all." We learn that the star Aldebaran " in Arabic means the Follower, since it follows the Seven Sisters round the sky." We learn how to use the Mathematicall Jewell: " whose purpose is, curiously enough, to avoid mathematics "; how to solve spherical triangles by drawing them on ping-pong balls, and how to construct a home-made astrolabe. Just to show that this really is a serious subject, the author in a concluding chapter outlines the conventional mathematical formulæ for solving the problems which he has been handling with such originality. Exercises with fully worked-out solutions are given at the end of the book.

Our only quarrel is with the sub-sub-title. The word " primer " suggests a text-book which is elementary and dull; this one may be simple but never fails to be amusing. And, though it may be bought in the first place by the Airman Cadet, we suspect that it will not be long before the said Cadet's father spends his crossword hour in making a quadrant and working out the various exercises for himself.

THEODORE VON KARMAN. ANNIVERSARY VOLUME. APPLIED MECHANICS. California Institute of Technology. 1941. $3.75.

Seldom has such a tribute, as exemplified in this volume, been paid to any man, and seldom has such a tribute been so well merited. The volume is a collection of scientific papers which have been contributed by a small number of Professor von Karman's friends to express their congratulations on his sixtieth birthday and to show their appreciation of his own remarkable work. In this volume a list of over ninety papers is given in varied fields of engineering activity.

Clark B. Millikan, in an introductory appreciation of von Karman's work draws attention to the three distinctive and characteristic elements of the work.

(*a*) The discovery and presentation of a new conception of some phenomenon which had hitherto remained quite unexplained and mysterious, in other words, creative scientific conception at its highest level.

(*b*) The classifying and reducing to clear and transparent form of material which had before been confused and terse only imperfectly comprehended.

(*c*) The finding of the essential physical elements in complicated engineering problems so that rational and simple approximate solutions can be obtained.

The presence of these elements in a scientific or technical work is evidence of quality of the highest order; the repeated occurrence of all these in the publications of one man is almost unique.

As a teacher von Karman has proved as outstanding as he has shown himself in his own work. His lectures are models of clarity, and he has inspired all his students with his own enthusiasm and methods of research, to the great gain of science in many directions.

The volume under review contains twenty-six papers of which the following are of direct interest to aircraft engineers:—

H. Bateman.—Some definite integrals occurring in aerodynamics.

Max M. Munk.—On the geometry of streamlining.

Theodore Theodorsen.—Impulse and momentum in an infinite fluid.

J. L. Synge and W. Z. Chien.—The intrinsic theory of elastic shells and plates.

S. Timoshenko.—The forced vibrations of tie rods.

Th. Troller.—A note on the effect of the wind tunnel size on pitching moments.

A. Nadai.—The creep of metals under various stress conditions.

L. H. Donnell.—Stress concentrations due to elliptical discontinuities in plates under edge forces.

Hars Reissner.—On lubrication flow with periodic distribution between prescribed boundaries.

R. v. Mises.—Some remarks on the laws of turbulent motion in tubes.

Wolfgang B. Klemperor.—Stress pattern crazing.

It is not possible in the space available to say more than this, that all the articles in the volume are by recognised experts and this, in itself, makes the volume not only one of real value, but one which is a great tribute to a great man.